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| **Probability**  Consider an experiment whose outcome is not predictable with certainty. However, although the outcome of the experiment will not be known in advance, let us suppose that the set of all possible outcomes is known. This set of all possible outcomes of an experiment is known as the **SAMPLE SPACE** of the experiment and it is denoted by S.  Example: If the experiment consists of flipping two coins, then the sample space consists of the following four points S = {HH, HT, TH, TT }  Each outcome in a sample space is called a Sample Point Number of sample points in a sample space S is **n(S) = nk** Where n = number of outcomes and k = number of objects  **Probability:**  If an experiment results in ‘n’ exhaustive, mutually exclusive and equally likely cases and ‘m’ of them are favorable to the happening of an event ‘A’ then Probability of happening of A is    Since the number of cases in which the event A will not happen is ‘n – m’, the probability that event A will not happen is:    Therefore  **Axioms of probability:**  Consider an experiment whose sample space is S. For each event E of the sample space S, then       **Laws of probability:**   1. Addition theorem:     If A and B are exclusive events i.e. disjoint sets, then:   1. Addition theorem (for three events):If A, B and C are pairwise exclusive events     Complementary Event**:** |
| **Conditional Probability and Independence**  If A and B are two events in a sample space S, then the probability of the event B when the event A has already occurred is called the conditional probability of B and is denoted by P(A|B) and defined as    The probability P(A|B) is an updating of P(A) based on the knowledge that event B has already occurred.  **Multiplication law of probability:**  **Independent events:**  A set of events is said to be independent if the occurrence of any one of them does not depend on the occurrence or non-occurrence of the others. If two events A and B are independent then:    **Theorem of total probability:**  If B­1, B2,……Bn be a set of exhaustive and mutually exclusive events and A is another event associated with Bi, then    **Baye’s theorem:**  If E1, E2, E3, . . . En are mutually exclusive and exhaustive events with P(Ei) ≠ 0 for i = 1 to n of a RANDOM experiment then for any arbitrary event ‘A’ of the sample spaces of the above experiment with P(A) > 0 , we have i=1 |
| **(Basic Probability)**   |  |  | | --- | --- | | Q.01 | If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7? | |  | 1/6 | | Q.02 | A fair coin is tossed 4 times. Define the sample space corresponding to this random experiment. Also give the subsets corresponding to the following events and find the respective probabilities.   1. More heads than tails are obtained. 2. Tails occur on the even numbered tosses. | |  | 1. 5/16, b. 1/4 | | Q.03 | If 3 balls are “randomly drawn” from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and the other two black? | |  | 4/11 | | Q.04 | A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women? | |  | 240/1001 | |
| **(Conditional Probability)**   |  |  | | --- | --- | | Q.01 | A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. What is the probability that the other one is also good? | |  | 5/9 | | Q.02 | Two fair dice are thrown independently. Three events A, B, and C are defined as follows:   1. Odd face with the first die. 2. Odd face with second die. 3. Sum of the numbers in the 2 dice is odd. Are the events A, B and C mutually independent? | |  | No. | | Q.03 | From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and multiplied. What is the probability that the product is positive? | |  | 505/1001 | | Q04 | A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen from the lot at random (without replacement). Find the probability that   1. Both are good. 2. Both have major defects. 3. At least 1 is good. 4. At most 1 is good. 5. Exactly I is good 6. Neither has major defects 7. Neither is good | |  | 3/8,1/120,7/8,5/8,1/2,91/120,1/8 | | Q.05 | There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times, what is the probability that the false coin has been chosen and used? | |  | 16/19 | | Q.06 | A bag contains 5 balls and it is known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white | |  | 1/2 | |

**Tutorial Questions**

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| Probability spaces | |
| Q.01 | A bag contains 8 white and 6 red balls. Find the probability of drawing of drawing two balls of the same color. |
|  | 43/91 |
| Q.02 | There are 4 letters and 4 addressed envelopes. If the letters are placed in the envelopes at random, find the probability that (i) none of the letters is in the correct envelope and (ii) at least 1 letter is in the correct envelope, by explicitly writing the sample space and the event spaces. |
|  | 1. 3/8 2. 5/8 |
| Q.03 | Find the probability of drawing an ace or a spade or both from a deck of cards. |
|  | 4/13 |
| Q.04 | There are 11 tickets in a box bearing numbers 1 to 11. Three tickets are drawn one after the other without replacement. Find the probability that they are drawn in an order bearing 1. Even, odd, even number, 2. Odd, odd, even number. |
|  | 4/33, 5/33 |
| Q.05 | If P(A)=0.3,P(B)=0.5, Find P(AorB) if A & B are exclusive and if A & B are independent |
|  | 0.8,0.65 |
| Conditional Probability And Independence | |
| Q.06 | A box contains 2 white and 4 black bals. Another box B contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white. |
|  | 16/39 |
| Q.07 | The students in a class are selected at random, one after the other, for an examination. Find the probability that the boys and girls in the class alternate if   1. The class consists of 4 boys and 3 girls. 2. The class consists of 3 boys and 3 girls. |
|  | 1/35, 1/10 |
| Q.08 | There are three bags; first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag. |
|  | 6/11 |
| Q.09 | If P(A)=1/4, P(B)=1/3 and P(A or B)=1/2, Evaluate P(A|B) and P(B/A) |
|  | ¼,1/3 |
| Q.10 | Three machines A, B and C produce identical items. Of their respective output 5%,4 % and 3% of items are faulty. On a certain day, A has produced 25% of the total output, B has produced30% and C the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output? |
|  | 0.3555,C |